08:32:45 OCA PAD INITIATION - PROJECT HEADER INFORMATION 05/24/88

Active

Project #: [redacted]
Center # : R6496-0A0

Cost share #: E-24-358
Center shr #: F6496-0A0

Rev #: 0
OCA file #:
Work type : RES
Document : CONT
Contract entity: GTRC

Contract#: N00014-88-K-0349
Prime #:

Mod #:

Subprojects ? : N
Main project #:

Project unit: ISYE Unit code: 02.010.124
Project director(s):
LLEWELLYN D C ISYE

Sponsor/division names: NAVY / OFC OF NAVAL RESEARCH
Sponsor/division codes: 103 / 025

Award period: 880601 to 890131 (performance) 890331 (reports)

Sponsor amount	New this change	Total to date
Contract value	54,616.00	54,616.00
Funded	54,616.00	54,616.00
Cost sharing amount		2,731.00

Does subcontracting plan apply ?: N

Title: LOCATION AND DISTRIBUTION OF LOCAL OPTIMA

PROJECT ADMINISTRATION DATA

OCA contact: E. Faith Gleason 894-4820

Sponsor technical contact

NEAL D. GLASSMAN, CODE:1111MA
(202)696-4362
MATHEMATICAL SCIENCES DIVISION, ONR
800 N. QUINCY STREET
ARLINGTON, VA 22217-5000

Sponsor issuing office

MICHELLE CARROLL, CODE:1511:MC
(202)696-4513
OFFICE OF NAVAL RESEARCH
800 N. QUINCY STREET
ARLINGTON, VA 22217-5000

Security class (U,C,S,TS) : U
Defense priority rating : N/A
Equipment title vests with: Sponsor
PRIOR APPROVAL REQUIRED.

ONR resident rep. is ACO (Y/N): Y
GOV'T supplemental sheet
GIT X

Administrative comments -
INITIATION.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Date 7/31/89

ct No. E-24-643 Center No. R6496-0A0

ct Director D. C. Llewellyn School/Lab ISyE

or Navy

act/Grant No. N00014-88-K-0349 GTRC XX GIT

Contract No.

Location and Distribution of Local Optima

tive Completion Date 1/31/89 (Performance) 3/31/89 (Reports

out Actions Required:

None
Final Invoice or Copy of Last Invoice
Final Report of Inventions and/or Subcontracts Already submitted
Government Property Inventory & Related Certificate Already submitted
Classified Material Certificate
Release and Assignment
Other

es Subproject No(s).

ject Under Main Project No.

ues Project No. Continued by Project No.

bution:

oject Director
ministrative Network
counting
ocurement/GTRI Supply Services
search Property Management
search Security Services

X Reports Coordinator (OCA)
X GTRC
X Project File
2 Contract Support Division (OCA)
Other

DEPARTMENT OF THE NAVY
CONTRACT N00014-88-K-0349

FINAL REPORT

Practioners have long been struggling to discover more effective heuristics for many important problems in combinatorial optimization (see e.g., Cornuejols, Nemhauser and Wolsey [1983], Dearing, Francis and Lowe [1976], Derigs [1985], Handler and Mirchandani [1979], Hornung [1983], and Tovey [1983]. Simulated annealing is a new approach to this problem, see e.g., Kirkpatrick, Gelat and Vecchi [1983], and Lundy and Mees [1986]). Finding local optima is at the heart of many of these heuristics.

This proposal is for a continuation of work currently being funded by ONR. The principal investigator has initiated a study of the location of local optima in discrete structures. To date, two papers have been written and accepted for publication. The first is "Finding Saddlepoints of Two-Person Zero-Sum Games," coauthored with Craig Tovey and Michael Trick, to appear in *The American Mathematical Monthly*. This investigates the problem on finding local and global saddlepoints of a matrix. Using fractal analysis we find that locating a local saddlepoint is a harder problem then the corresponding one for a global saddlepoint. The second paper is entitled "Local Optimization on Graphs," coauthored with Craig Tovey and Michael Trick, to appear in *Discrete Applied Mathematics*. This work uses game theory to develop a theoretical base from which lower and upper bounds on the amount of work required to find a local optima may be gleaned. Another paper, "Dividing and Conquering the Square," to be coauthored with Craig Tovey, is in the final stages of research. This will look at the problem of local optimization on grid graphs.

We are now studying different triangulations of graphs. This is a natural next step for two reasons. First, triangulated graphs are closely related geometrically to grid graphs and hence we hope that our geomtric algorithms will extend in this new arena. Perhaps more importantly, triangulations were historically used for optimization over tabular functions. (In particular, Delauney triangulations, see e.g., Chang and Lee [1984], and Lee [1985]). There has been a significant amount of research into the structure of triangulated graphs (see e.g., Syslo [1985], Hayward [1985], and Katajainen and Nevalainen [1986]), but to date noone has used these in the search of local optima for discrete functions.

Our attention is at present concentrated on "simple" graphical structures. This is for two main reasons - first, we expect our techniques and theories to extend quite nicely to other, more complicated structures. Also, as history shows in the arena of continuous optimization, real problems rarely fall into the classes of pretty textbook problems. In combinatorial optimization this is also true and so only studying the problems that give rise to the well-known well-formulated NP-Complete problems is not realistic. Rather, a theory that helps to solve a general discrete problem is more useful. Our work will give us information about what types of simple to describe neighborhood structures give rise to efficient local optimization procedures. This in turn will be used to develop methods to define neighborhood graphs for general problems that will allow us to solve them. Eventually, it will be interesting to check if these neighborhood structures are actually the ones that the textbooks suggest for the classical problems.

With techniques ranging from game theory to fractal analysis we propose to continue to determine the complexity of finding local optima in discrete structures as well as to investigate the behavior of these solutions. If a heuristic involves determining the location of local optima, then this local optimization subroutine must be efficient for the algorithm to be effective. Hence, it is important to study lower bounds on the work needed by local optimization techniques and to investigate methods that approach these bounds. Moreover, information about the interrelationships of the local optima, i.e. more knowledge of where the local optima are located, how their values are distributed, and how these factors relate to the corresponding information about the global optima and their values, will permit a scientific approach to the development of local optimization algorithms. Developing a rigorous mathemathical approach to this problem is new. While many practioners report on different techniques in discrete local optimization, there is neither a consensus on what works well in practice, nor a strong theoretical foundation describing the relative complexities and benefits of these different methods. Currently, procedures are written using only intuition and past empirical experience. In addition, any knowledge of local optima in discrete structures will improve our understanding of these structures and hence aide in our search for global optimization techniques.

The primary method currently employed in local optimization algorithms is local improvement. This technique involves pivoting from a solution candidate to a nearby candidate that has a better function value. Given the pivoting rule, a neighborhood structure of the candidate solutions is imposed. In preliminary investigations, we studied bounds on the worst case amount of work needed by any type of algorithm to find local optima in particular neighborhood structures. This is one of the first studies to rigorously investigate techniques other than local improvement. The mathematics used here includes analyzing a two person game and finding the fractal dimension of a collection of entries in a matrix. We are anxious to continue to pursue this and to look for other neighborhood structures which permit this type of analysis. Further, we wish to begin to address the larger questions of what makes an efficient local optimization algorithm and what is the complexity of this problem in general. (Note that Johnson, Papadimitriou and Yannakakis [1985] have initiated investigation into this latter question).

We have begun by studying several basic discrete problems; in particular, problems where the neighborhood structure is easy to analyze such as those that form grid graphs or planar graphs. Here we have found bounds on the amount of work involved in locating local optima. We are investigating information about the distributions of such solutions and their functional values; as well as looking at other simple settings such as triangulated graphs. Once we have a better understanding of these elementary settings, we will move on to more important problems in combinatorial optimization, seeking to answer the same questions. While looking at these specific problems, we will aim to form a coherent approach and analysis of the subject of local optimization in discrete structures. If successful, this will help to locate structures for which the information gleaned from the local optimization process is useful in the pursuit of global optima. In addition, the complexity analysis of the local algorithms will serve to disqualify certain general types of global algorithms as inefficient. More specifically, if we obtain a lower bound on the worst case amount of work involved in locating a local optimum, then clearly any global optimization algorithm must also have this as a lower bound. Further, if we find that certain genres of local optimization algorithms do not perform near this lower bound, then probably these will not extend to efficient global methods. It should be noted that any progress toward more efficient solution techniques for discrete optimization problems will provide an enormous boost to the current knowledge base in all areas serviced by integer programming, including but not restricted to plant location, production planning and job shop scheduling.

In section 2 below, we define the general combinatorial optimization problem and the necessary notation for the rest of the proposal. The problem of finding local optima is discussed in section 3. In section 4 we outline the plan for our research in this area.